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Boundary-Layer Effects on Pressure Variations in Recovery Tube

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FOR generating high Reynolds number transonic flows, the application of the Ludwieg tube principle has been considered to be an attractive proposition because of its simplicity and also its high flow quality. To improve the efficiency of operation of the basic Ludwieg tube, the addition of a long tube called the recovery tube has been suggested.[1] While the calculation procedure for the ideal flow quantities in the recovery tube is simple, deviations from these calculations can be expected because of the real gas effects. The purpose of this paper is to present an analysis to calculate the departure from the ideal flow behavior due to the turbulent boundary-layer growth on the tube wall. The analysis, based on the solution of one-dimensional unsteady flow equations with distributed mass sources, predicts that, at the tube entry, the pressure increases with time due to boundary-layer growth. This can cause early termination of the uniform test section flow by unchoking of the throat. The agreement with the limited available data on pressure measurements[2] indicates that the analysis can be used with confidence to obtain preliminary estimates of the boundary-layer effects in the recovery tube for larger tunnels like that proposed for the European Transonic Ludwieg Tube Tunnel.[1]

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Analysis

The flow into the recovery tube is idealized by inflow through a choke into a long tube closed at one end (Fig. 1a). The pressure inside the tube is kept below the ambient pressure by a diaphragm attached to the nozzle end. The rupture of the diaphragm initiates the propagation of a shock wave and a following contact surface into the tube. In this analysis, it is assumed that the nozzle throat is choked and the deceleration of the flow to subsonic Mach numbers in the tube (region 3) occurs through a stationary shock located in the diffuser section. This phenomenon is similar to that happening in the recovery tube of transonic tunnel.[1] Across the contact surface, the pressure and the velocity are continuous but the temperature has a jump since region 2 ahead of it contains the gas processed by the shock wave. The boundary-layer growth in the tube is considered in two parts. Firstly, the unsteady boundary layer in region 2 induced by the passage of the shock wave. Secondly, for region 3 between the tube entry and the contact surface, a steady flat plate type boundary layer starting from the entry section of the tube from the instant the constact surface has passed is assumed. The downstream edge of this boundary layer is assumed to move along with the contact surface and at any station in between, the boundary-layer growth is considered time independent. For region (2) induced by the shock wave, the unsteady boundary-layer growth is calculated by considering the equivalent steady-state problem with shock-fixed coordinates.[3]

For calculating the perturbation in the flow quantities, the boundary-layer growth is assumed to act as distributed mass sources the strength of which is dependent on the vertical velocities induced at the outer edge of the boundary layer. For the one-dimensional flow in a tube of uniform cross section with distributed mass sources, the solution of the governing differential equations can be written as[4]

$$\Delta p^{\pm} = \pm \frac{2\gamma p}{ad(1 \pm M)} \int_{\mp\infty}^{\kappa} v\left(\xi, t - \frac{\kappa - \xi}{a-u}\right) \mathrm{d}\xi \tag{1a}$$

$$\Delta p = \Delta p^{+} + \Delta p^{-} \tag{1b}$$

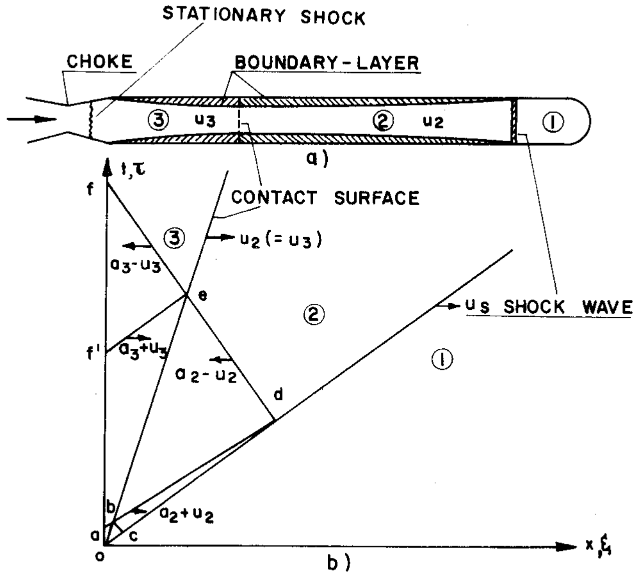

Fig. 1 Flow in recovery tube with boundary-layer development and $x-t$ diagram.

where p is the pressure, γ the ratio of specific heats, a the local speed of sound, M the local Mach number assumed subsonic, d the tube diameter, v the vertical velocity induced at the boundary-layer edge and Δ the perturbation. The superscripts + and − denote the right and left running waves respectively. The disturbances introduced by the boundary layer travel both upstream and downstream. To calculate the pressure variation at any station, it is necessary to consider the contribution of both the left running and right running characteristics signifying the wave phenomena. For the variation at the tube entry, which is of present interest, the characteristic lines over which the integration of Eq. (1a) is to be carried out is shown in the $x-t$ diagram (Fig. 1b). However, for the case of weak shocks the major contribution comes from the characteristic lines *fe, ed* and *f′e*. It is found that the boundary layer developing between the nozzle entry and the contact surface gives rise to compression waves which cause the pressure to increase with time. However, the waves generated by the shock induced boundary layer (Region 2) are of opposite character which tend to reduce the pressure at $x=0$. The latter effect is small and hence there is a net increase in pressure with time at the tube entry due to the unsteady boundary-layer growth. For weak shock waves, the reflections of the right running waves at the contact surface and at the shock wave are of a lesser order of magnitude and can be neglected. However, the reflection at the tube entry, assumed choked, needs to be taken into consideration. A simple formula is derived for the strength of the reflected wave by equating the mass flow before and after the reflection. The strength of the reflected wave at f, denoted by Δ^+_{3f} can be written in terms of the strength incident wave, Δ^-_{3f} as

$$\Delta^+_{p,3f} = \Delta^-_{3f}(1-M_3)/(1+M_3) \tag{2}$$

where M_3 is the flow Mach number in region 3.

Results and Discussion

The results of the present analysis, calculated using the 1/7th power law velocity profile and the skin friction law $(\tau/\rho u^2) = k\ (\nu/u\delta)^m$ with $k=0.0225$ and $m=0.25$ for the turbulent boundary layer, are compared with the measurements of Ref. 2 in Fig. 2. The calculated time-wise variation of pressure at $x=0$ agrees well with the measurements. The effect of the total pressure loss in the entry is compared with measurements in Fig. 3. As β, defined as the ratio of total pressures at the diffuser entry and exit, increases, the pressure rise because of the boundary-layer growth also increases since the Mach numbers in the tube are correspondingly higher. When the nozzle throat is choked, the β and the flow Mach number M_3 are related by (for $\gamma = 1.4$)

$$(A/A_t) = \beta(1+0.2M_3^2)^3/1.728M_3 \tag{3}$$

where A and A_t are the tube and throat areas respectively. In Fig. 3, the experimental data for low values of β has been excluded since it is suspected that the nozzle throat might be unchoked for these values. These results suggest that the pressure build-up $(\Delta p_3/p_3)$ increases slowly with β.

The basic limitation of the analysis is the discontinuity in the boundary-layer profile, similar to that in Mirels' analysis[4] for shock tubes, because of the different theories used for calculating the boundary layer in regions 2 and 3. For weak shocks, the discontinuity is small. The assumption of a thin boundary layer is not satisfied for large time when the boundary-layer thickness is comparable to the tube radius. In spite of this, the agreement between theory and experiment is good. Since the freestream conditions are approached in an asymptotic manner, it may be expected that the boundary-layer theory used may overestimate the thickness.

For the present method to be useful, it is necessary that the configuration shown in Fig. 1 completely describes the flow in the recovery tube of a Ludwieg tube test facility.[1] Since the

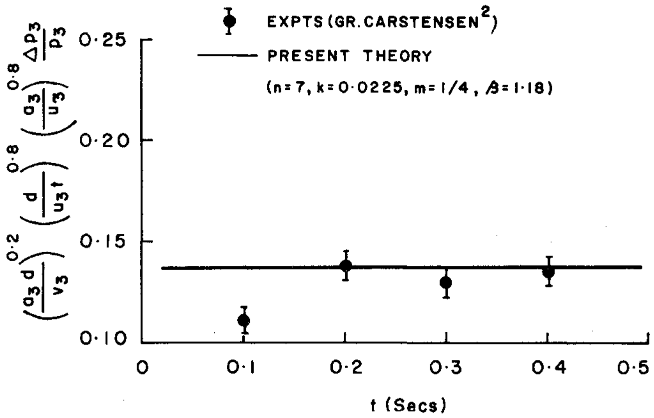

Fig. 2 Comparison of theory with experimental results for the pressure variation with time in the recovery tube.

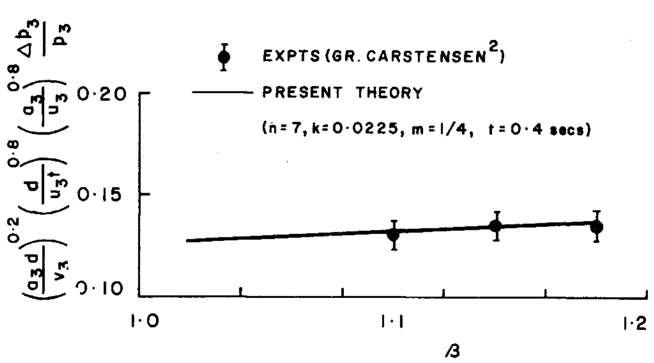

Fig. 3 Effect of total pressure loss factor β on pressure perturbation at $x=0$.

flow is choked, the computed perturbations in the recovery tube do not affect the test section in the charge tube. In practice, the pressure build-up in the recovery tube causes the assumed stationary shock in the diffuser to weaken and move upstream and eventually unchoke the throat affecting the test section flow. This process is nonlinear and is not considered in the present method. However, since during this nonlinear pressure build-up process the shock pressure loss is decreasing, the effective value of β will also correspondingly reduce with the consequent effect of decreasing the rate of pressure build-up (Fig. 3). Further, the unsteadiness of the boundary layer has also been neglected in the present method. In spite of these limitations, the reasonably good agreement of the method with the experiments suggests that the analysis will be helpful in fixing the upper and lower bounds of pressure variations depending on the estimated values of β in the initial design studies.

The charge tube boundary layer, not taken into account in the analysis, causes time-wise variation of the total pressure in the test section. At least for low Mach numbers in the charge tube this effect can be kept small as shown by Piltz.[5] However, this can be calculated by the methods of Refs. 4 and 6. More detailed comparisons for the attenuation of the shock wave and calculations for the Large Transonic Tunnel[1] are given in Refs. 7 and 8.

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Technical Comments

Comment on "Measurements in the Laminar Near-Wake of Magnetically Suspended Cones at $M_\infty = 6.3$"

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THE gap at low hypersonic Mach numbers in the data on interference-free, laminar near-wakes has been well filled by the extensive survey of Blankson and Finston.[1] However, in discussing earlier work, they have not referred to preliminary measurements made using the Royal Aircraft Establishment magnetic suspension system[2,3] in 1967-8. These were mentioned by Crane in Ref. 4 and the full details form part of a thesis.[5] Subsequently, the correspondent's namesake[6] investigated more fully the extent of interference to the flow pattern resulting from probe stems, a point which Blankson and Finston did not discuss.

These measurements were made behind sharp, 10° half-angle cones at $M_\infty = 8.5$ in a blow-down tunnel, the jet having a uniform core around twice the diameter of the cone base. In addition to a flat-base cone, a model with a partially rounded base was tested, the corner being rounded to a radius of half the original base radius. The short period of time between model launch and recapture (around ½ minute) was one factor limiting the amount of data which could be obtained, also preventing data reduction to give the primary flowfield variables. Freestream stagnation pressure and temperature were 2.86 MPa and about 660K, giving a freestream Reynolds number of 312,000 based on the 50.8 mm cone base diameter. The model wall temperature (determined by thermocouples in separate tests) varied during and between tunnel runs, and also varied over the surface in a different manner for each model; values of $T/T_{0\infty}$ for the conical surface ranged from 0.57 to 0.65, while values for the base were estimated to be typically 0.09 lower than these.

Pitot tubes sufficiently large to give an acceptable response time (outer diameter 0.03D or larger) were mounted on a transverse wedge-plus-afterbody stem such that the probe head moved in an arc of radius 2D passing through the wake centerline. On the basis of the data of Zakkay and Cresci (Ref. 2 of Blankson and Finston), it was assumed that these tubes were not subject, in the region of use, to large errors from viscous effects which can occur at probe Reynolds numbers below about 150. Equilibrium hot-film temperatures were measured using conical probes; the data were adjusted to a model surface temperature of 0.60 times the freestream stagnation temperature by an approximate procedure involving repetition of a series of tunnel runs with a reversed sequence of probe positions, but the lack of an adequate range of flow conditions for calibration limited the value of these results.

Blankson and Finston's observations on the merging of the lip and wake shock waves at hypersonic M_∞, and on the location of the wake shock origin near $X/D = 0.8$, confirm the evidence of schlieren photographs in Ref. 5, from which Fig. 1 was sketched. A lip shock was not detected by the schlieren or the coarsely-spaced probe measurements, but the orientation of the shock behind the rounded base suggests a continuous lip and wake shock structure.

Some pitot pressure profiles from Ref. 5 are reproduced in Fig. 2. The estimated error in the quantity $10^3 p_p/p_{0\infty}$ is ± 0.02 for r/D less than about 0.25 and ± 0.1 elsewhere. Perpendicular traverses to check the symmetry of the flowfield were not possible, and reliance was placed on mechanical prealignment of the model with the nozzle to give a zero angle of attack. At $X/D = 1.5$, a small amount of probe stem interference was noted, causing a 2 to 3% reduction in drag (measured by electromagnet current) as the probe covered its "radial" traverse; measurements at $X/D = 1.0$ were possible with a longer probe head. Axial movement of the probe over the measuring range, with the head on the wake axis, did not cause any detectable change in drag. Schlieren photographs indicated that at $X/D = 1.5$, the probe radial position also affected the local wake shock diameter, but the axial position did not affect the shock diameter measured at $X/D = 2$. Later observations,[6] with the tip of a probe stem positioned on the axis, indicated a reduction in the diameter and a downstream movement of the wake shock source on the photographs as the stem approached the rounded-base model. Location of the shock source is somewhat subjective and the measurements showed con-

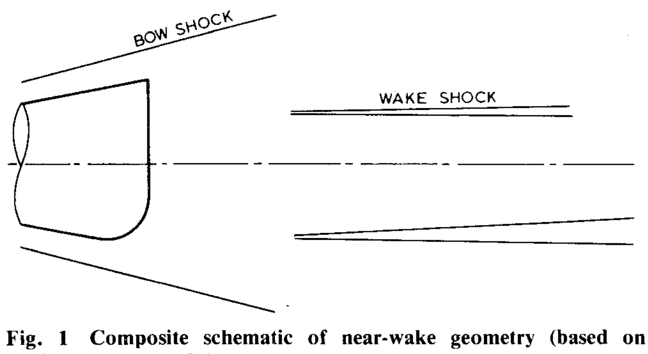

Fig. 1 Composite schematic of near-wake geometry (based on schlieren photographs).

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